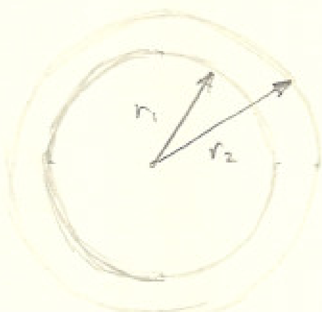
The spherical shell w constant surface charge density

$$\vec{E} = \begin{cases} \frac{Q_{net}}{4\pi\varepsilon_0} \frac{\hat{r}_p}{r_p^2} & \text{if } r_p > R \\ 0 & \text{if } r_p < R \end{cases}$$

Imagine a solid sphere w constant volume charge density $\rho(r)$



hollow sphere divided into thin spherical shell w thickness dr.

Sphere is broken into thin shells so so $P(r)$ is constant on the shell.

$$\vec{E}_1 = \frac{Q_1}{4\pi\varepsilon_0} \frac{\hat{r}_p}{r_p^2}$$

$$\vec{E}_2 = \frac{Q_2}{4\pi\varepsilon_0} \frac{\hat{r}_p}{r_p^2}$$

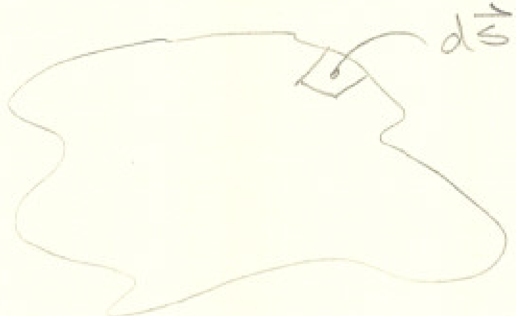
$$\rightarrow \vec{E} = \frac{Q_1 + Q_2}{4\pi\varepsilon_0} \frac{\hat{r}_p}{r_p}$$

Flux of a Vector Feild

scaler $\rightarrow$ $\Phi \equiv \int_S \vec{E} \cdot d\vec{S}$ Electric feild. surface element

$dA = dx\,dy$

$dA = r_s\,dr\,d\theta$



Closed Surface:

defined interior and exterior

Open Surface:

neither ie: paper.

note: the direction of $d\vec{S}$ is perpendicular to surface.

Suppose a charge $+Q$ is at the origin. What is Φ_E through a spherical surface of radis R. $\vec{E}$ is everywhere radial.

$$\vec{E} \sim \vec{r}_p$$

$\vec{E} \cdot d\vec{S}$ at any point $\vec{r}_p$ is:

$$|\vec{E}(\vec{r}_p)|\, R^2 \sin\theta\, d\theta\, d\phi$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0} \frac{\hat{r}_p}{r_p^2} \qquad d\vec{S} = R^2 \sin\theta\, d\theta\, d\phi\, \hat{r}_p$$

$$\vec{E} \cdot d\vec{S} = \frac{Q}{4\pi\varepsilon_0} \frac{1}{R^2} R^2 \sin\theta\, d\phi$$

$$\oint \vec{E} \cdot d\vec{S} = \int \frac{Q}{4\pi\varepsilon_0} \sin\theta\, d\theta\, d\phi = \frac{Q}{4\pi\varepsilon_0} 4\pi = \frac{Q}{\varepsilon_0}$$

note: this holds true for any geometry.